

N78-11003

NASA TM-75059

EFFECT OF FLAP DEFLECTION ON THE LIFT COEFFICIENT OF WINGS  
OPERATING IN A BIPLANE CONFIGURATION

J. Stasiak

~~CASE FILE~~  
~~COPY~~

Translation of "Wplyw wychylenia klapy na wspolczynniki sily  
nosnej platow pracujacych w układzie dwuplata," Prace  
Instytutu Lotnictwa, No. 64, 1976, pp. 5-28



|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |                                                      |                                                          |           |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------|----------------------------------------------------------|-----------|
| 1. Report No.<br>NASA TM-75059                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | 2. Government Accession No.                          | 3. Recipient's Catalog No.                               |           |
| 4. Title and Subtitle<br>EFFECT OF FLAP DEFLECTION ON THE LIFT<br>COEFFICIENT OF WINGS OPERATING IN A BI-<br>PLANE CONFIGURATION                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |                                                      | 5. Report Date<br>November 1977                          |           |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |                                                      | 6. Performing Organization Code                          |           |
| 7. Author(s)<br>J. Stasiak                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                                                      | 8. Performing Organization Report No.                    |           |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |                                                      | 10. Work Unit No.                                        |           |
| 9. Performing Organization Name and Address<br>Leo Kanner Associates<br>Redwood City, California 94063                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |                                                      | 11. Contract or Grant No.<br>NASW-2790                   |           |
|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |                                                      | 13. Type of Report and Period Covered<br><br>Translation |           |
| 12. Sponsoring Agency Name and Address<br>National Aeronautics and Space Adminis-<br>tration, Washington, D.C. 20546                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |                                                      | 14. Sponsoring Agency Code                               |           |
| 15. Supplementary Notes<br><br>Translation of "Wplyw wychylenia klapy na wspolczynniki sily<br>nosnej platow pracujacych w układzie dwuplata," Prace Insty-<br>tutu Lotnictwa, No. 64, 1976, pp. 5-28                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                                                      |                                                          |           |
| 16. Abstract<br>The work was carried out on the basis of needs which emerged<br>while working out the construction of an agricultural air-<br>craft in a biplane configuration with equipment for increasing<br>lifting power.<br>Experimental studies carried out in an aerodynamic tunnel on<br>biplane models with a lift flap permitted the determination<br>of the effect of flap deflection on the aerodynamic coeffi-<br>cient of the biplane as well as of the individual wings. In<br>addition, optimization of the position flap was carried out,<br>and the effect of changes in the chord length of the lower<br>wing was determined for the aerodynamic structure of a bi-<br>plane with a lift flap on the upper wing. |                                                      |                                                          |           |
| 17. Key Words (Selected by Author(s))                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                                                      | 18. Distribution Statement<br><br>Unclassified-Unlimited |           |
| 19. Security Classif. (of this report)<br>Unclassified                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               | 20. Security Classif. (of this page)<br>Unclassified | 21. No. of Pages<br>35                                   | 22. Price |

## SYMBOLS

- S - Surface of the biplane cellule equal to the sum of the upper and lower wing surfaces:  $S = S_u + S_l$
- b - Wing span of the biplane
- $l_u$  - Geometric chord of the upper wing
- $l_l$  - Geometric chord of the lower wing
- $\lambda$  - Geometric elongation
- $\lambda_{ef}$  - Effective elongation
- h - Spacing of airfoil
- $\beta$  - Leading angle of the wings
- $\sigma$  - Angle of wing inclination
- $\kappa$  - Ratio of corresponding dimensions of edge panels
- v - Rate of nonturbulent flow
- q - Dynamic pressure
- $\alpha$  - Angle of attack
- $\Gamma$  - Rate circulation
- $\rho$  - Density
- TF - Coefficient of tunnel turbulence
- $R_e$  - Reynolds number
- $Re_{ef}$  - Effective Reynolds number
- Cx - Coefficient of resistance
- Cz - Lift coefficient
- Cm - Coefficient of bending moment
- $\Delta C_{z_{int}}$  - Increment in lift coefficient, caused by mutual operation of the wings, determined as a function of the angle of attack at  $\delta_k = \text{constant}$
- $\Delta C'_{z_{int}}$  - Increment in lift coefficient, caused by mutual operation of the wings, determined as a function of angle of flap deflection at  $\alpha = \text{constant}$
- $\Delta C_z$  - Increment in the lift coefficient, caused by flap deflection
- $\delta_k$  - Angle of flap deflection at the upper wing (for the case when the flap is located only on the upper wing)
- $\delta_{ku}$  - Angle of flap deflection on the upper wing (for the case when the flap is located on both wings)
- $\delta_{kl}$  - Angle of flap deflection on the lower wing (for the case when the flap is located on both wings)

# EFFECT OF FLAP DEFLECTION ON THE LIFT COEFFICIENT OF WINGS OPERATING IN A BIPLANE CONFIGURATION

J. Stasiak  
Prague Aeronautical Institute

## 1. Introduction

16\*

### 1.1. Origin and Subject of Work

The work below was undertaken as a result of needs which arose while working out the construction of an agricultural aircraft, whose lift device is biplanar, equipped with devices for increasing lift, namely a slot and double-aperture flap located on the upper wing.

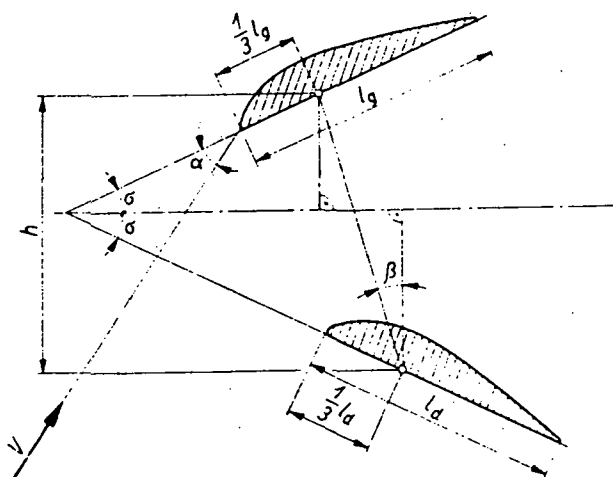


Diagram for symbols

It was presumed that flap deflection causes a deterioration of the aerodynamic properties of the lower wing, and especially of the lift coefficient. With the aim of verifying this presumption, experimental studies were ordered conducted in an aerodynamic tunnel on a segment of the biplane cellule, having as purpose the determination of the mutual effect of the wings during double-aperture flap deflection. In addition, the ef-

fects obtained from the studies were to be used to ascertain the optimal cell for the lifting biplane with flap deflection.

The studies conducted involved:

1. determination of the effect of deflecting a flap located

\* Numbers in the margin indicate pagination in the foreign text.

on the upper wing,

2. determination of the effect of deflecting a flap located on the lower wing,
3. determination of the effect of flaps located on the upper and lower wings during simultaneous deflection of both flaps at the same angle,
4. determination of the effect which exerts a decrease in the chord length of the lower wing, in comparison with the chord of the upper wing, with deflection of flaps located on the upper wing.

On the basis of the effects of the studies involved in points 2, 3, and 4, conclusions were drawn for the optimal biplane configuration with deflected lift flaps.

#### 1.2. State of the Studies in the Biplane Domain

In considering the existing requirements, the question of mutual operation of wings in a biplane configuration and the associated effect on the aerodynamic coefficients of the biplane were studied very early, for in 1914 A. Betz [1 and 2] completed the first work touching upon this question. Since that time, many works have been completed in the area of biplane aerodynamics, which are divided into four basic groups.

Generally speaking, it is possible to say that works belonging to each of the four groups occupy above all a designation of the size of the lifting force acting on individual wings. The work belonging to the first group permitted, moreover, the determination of the induced resistance of the biplane and load distribution along the length of the wingspread. /7

The first group included theoretical work involving three-dimensional streamlining of the biplane.

The second group represents theoretical work concerned with flow around the biplane as a two-dimensional object. In studies of this sort are the theory of true angle reciprocation or integral calculus.

The third group consists of work of an experimental character. Work belonging to this group includes studies in aerodynamic tunnels of wing configuration or studies of aircraft in flight. Systematic results of this sort of study are given in NASA Technical Notes No. 310, 325, and 330 and NASA Report No. 417.

Load measurements concern the force and effective moment for the individual wings in a biplane configuration and for the whole biplane configuration. Measurements were made for many deflections using pressure distributions in the individual sections, with which a normal force was obtained based on corresponding treatment.

It must be noted that all the experimental studies were conducted in tunnels of not very great dimensions for the spatial measurements ( $\phi = 1.5$  m) and for not very high speeds ( $\sim 30$  m/sec) and length of the geometric wing  $\lambda = 6$ . As a result, very small Reynolds numbers were obtained ( $Re = 100,000-200,000$ ), which indicates that for such results obtained on the basis of the studies conducted, one must proceed with much caution.

In the fourth group of works belongs the question of biplane flow, works containing examples recounted and elaborated on the basis of the results obtained from previously executed theoretical and experimental works.

On the basis of the examples given in these works, it is possible to calculate the lift force operating on the individual wings by an assumption, which is called the total lift force operating on a biplane configuration.

## 2. Experimental Studies

### 2.1. General Assumptions for the Program of Studies

The problem of determining the mutual effect of the wings on the aerodynamic characteristics is explicitly constituted during analysis of the size of aerodynamic coefficients of the biplane cellule for a projected agricultural aircraft. Considering the lack of publications, on one hand, and, on the other hand, the limited technical possibilities for available measuring equipment which could be extended to studies of this sort, it was decided to conduct the studies in an aerodynamic tunnel using load measurements and studies of pressure distribution on the segment of the biplane cellule with the flap.

To adequately obtain large Reynolds numbers, it was decided to use a large-chord model of the biplane cellule:  $l_u = 338$  m. However, the wing span was limited ( $b = 800$  m), and therefore the model was also equipped with edge panels.

The choice of size for the edge panels for the segment of biplane cellule could be the subject of a separate, independent work, and therefore in order not to excessively complicate the question, the dimensions of the basic wings were chosen in a manner relating to the technical conditions of the tunnel.

The load measurements included, first of all, studies of each /8 wing separately from the edge panels, without the presence of a second wing. These studies will be called the studies of isolated wings. Secondly, the load measurements included studies of the biplane cellule as a whole. This permitted establishing the effect of total interference in the biplane cellule on the size of the aerodynamic coefficients, for the whole biplane cell, but it did not permit the determination of changes in the aerodynamic coefficients of each wing separately on the effect of wing interference

In order to establish this effect, measurements were made of pressure distributions on the isolated lower wing profile, and finally of the wing located in the biplane cell. By this means, the increment in lift coefficient  $\Delta C_{z_{int}} = f(\alpha)$  at  $\delta_k$  a constant for the lower wing, which caused interference of the upper wing. Nevertheless, on the upper wing with respect to the small dimensions of the flap, but especially the flap slot and wing slot, it was not possible to locate the corresponding number of ducts for pressure measurements. In this connection, measurements of pressure distributions dealt with on the upper wing remained as if approximated and had only an auxiliary task during analysis of the study results.

In order to determine the optimum biplane cellule, the possibility was anticipated of doing studies with the lower wing on a decreased chord and studies with altered lower-wing position relative to the upper wing.

The effect on the aerodynamic coefficients of a biplane cellule with such parameters remaining as dispersion, leading and trailing of wings was taken as known in principle, and therefore changes in these parameters were not anticipated, but not to excessively develop the program of studies.

## 2.2. Methods and Study Arrangements

Load measurements were done with the help of a load of three-component, simultaneous, auxiliary measurements of three components, namely the lift force, leading resistance, and bending moment. The measurements were in general done one time because the measurements which were based on or had a fundamental significance for the essential conclusions were repeated twice and even three times, which permitted avoiding accidental measurement errors. The long dimensions of the model with an aperture flap on the one hand, and the limited technical possibility of the tunnel and load on the



other, were the reason (in the case of slot deflection) for the very large loads aerodynamically. Moreover, for large angles of attack, vibration of the model was established for the effect of flow detachment, which was a great difficulty in making measurements in the neighborhood of  $C_{z_{max}}$ , but especially with regard to flow detachment. Therefore, also in order to avoid errors, measurements in the vicinity of  $C_{z_{max}}$  were likewise done many times and the average values were determined.

Measurements of pressure distributions were done using a sixty-point battery of manometers, which made possible simultaneous measurement of pressures along the profile. The results of the pressure-distribution studies were registered by means of filming the indications of the manometer battery, and finally were calculated with the corresponding enlarged print of the photographic indication for individual battery ducts. Treatment of the measured results of load as well as pressure-distribution measurements were completed using a ZAM-2 GAMMA or Odra 1204 mathematical calculator.

All the measurements, both load and pressure distribution, were done in a tunnel with open spatial measurements of 1.5 m in diameter and 2.2 m long. The speed within this tunnel could be changed within the limits of 15-40 m/sec.

The coefficient of turbulent streaming in the confines of the tunnel, measured by Dryden's method of measuring the pressure differences on a sphere, amounted to  $TF = 1.425$ .

### 2.3. Description of the Model

/9

A schematic of the segment established for a biplane cellule with edge panels, for which studies were done in an aerodynamic tunnel, is presented in Fig. 1a and b. The corresponding wings of the model are shown made from wood. The slot and the flap with slot, located on the upper wing were made of metal.

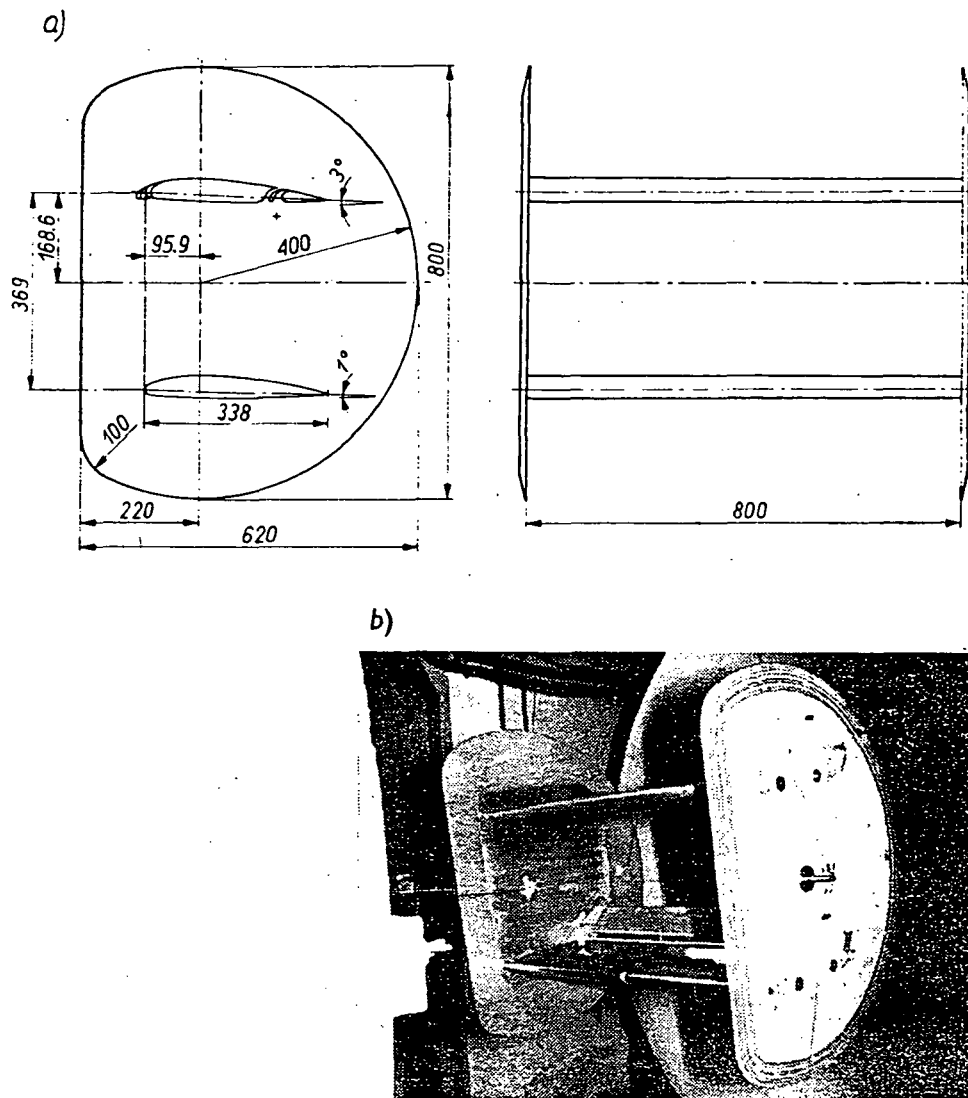


Fig. 1. Model of the lift cell of the biplane studied; a) construction sketch b) photo of executed model within the tunnel.

Construction of the model of a biplane cellule made possible studies of corresponding wings without the presence of a second one and exchange of the lower wing for another flap or a reduced chord. Such a solution established the sequence of the adopted program of studies, and moreover the "tactics" of the studies which aimed to obtain the information present with a minimal number of models.

The upper wing, in all the cases studied, had the profile CAGI P-II-14 with load length = 338 mm and span  $b = 800$  mm. The lower wing had the same geometric parameters which depended on the study program realized. Changes in the percentage of thickness of the lower-wing profile from the considerations used were chosen such that the absolute thickness of the profile remained almost without change. Moreover, the construction of the model made possible the displacement of the lower wing about the reduced chord  $l_l = l_u$  in two positions which are called the forward position and the rear position. The forward position of the lower wing is that position in which the attacking edge of both wings is in the same perpendicular plane. The rear position of the lower wing is that position in which the same perpendicular plane is the flow edge of both wings. /10

For all the pre-study configurations of the biplane, the trailing angle of the wings is  $\sigma = 2^\circ$ .

#### 2.4. Systematics and Study Results

The aerodynamic measurements taken in accordance with the determination in point 2.1 assumed for the program of studies can be divided into six basic groups, namely:

1. studies of the effect of flap deflection on the upper wing with closed and open slot: the upper and lower wing for the same profile CAGI-P-II-14 and the same chord  $l_l = l_u = 338$  mm,
2. studies of the effect of flap deflection on the lower wing; the upper and lower wing with the same profile CAGI and the same chord as in point 1,
3. studies of the effect of simultaneous flap deflection on upper and lower wings; upper and lower wing with the same profile and chord as in point 1,
4. studies of the effect of flap deflection on the upper wing for a reduced chord at the lower wing and equal

positions relative to the upper wing, namely in the forward and rear positions. The lower wing had the following profile and chord length:

-- profile NACA 2418; chord  $l_1 = 0.75 l_u$

-- profile NACA 2424; chord  $l_1 = 0.5 l_u$

5. studies of the effect of flap deflection on the upper wing on the lower-wing coefficients on the basis of pressure-distribution measurements.

The lower wing studied was in the forward and rear positions and had the following profile and chord length:

-- profile CAGI-P-II-14; chord  $l_1 = l_u$

-- profile NACA 2418; chord  $l_1 = 0.75 l_u$

-- profile NACA 2424; chord  $l_1 = 0.5 l_u$ .

The combination of all the configurations which were studied in the individual groups is given in Tables 1 and 2.

The measurements in groups 1 and 2, done using loads as an illustration, are both a more useful application in a biplane configuration to a flap on the upper wing and on the lower wing.

The measurements in group 3 were done likewise using loads to confirm the author's hypothesis that in a biplane configuration it is a useful application of deflected flaps on the upper and lower wings to prevent too large a decrease in lift force on one of the wings, due to interference.

Measurements in groups 4 and 5 were done likewise using loads and a battery of manometers which records pressure distributions, in order to determine the "optimalization direction" of a biplane cell with the flap located on the upper wing.

The studies of isolated wings mentioned in the individual groups of Tables 1 and 2 must be understood as studies of the

upper and lower wings, with edge panels, without the presence of a second wing.

TABLE 1

/11

| Group | Problem                                                                                                                         | Study Configuration | Slot   | Profile and chord length of lower wing | Flap De-flection |
|-------|---------------------------------------------------------------------------------------------------------------------------------|---------------------|--------|----------------------------------------|------------------|
| 1     | Determination of effect of flap deflection and open slot on upper wing on aerodynamic coefficients of biplane cellule           | Biplane cellule     | Closed | $l_l = l_u$                            | 0-50°            |
|       |                                                                                                                                 | Isolated upper wing | Open   |                                        | 0-50°            |
|       |                                                                                                                                 | Isolated lower wing | --     |                                        | --               |
|       |                                                                                                                                 |                     |        |                                        |                  |
| 2     | Determination of effect of flap deflection on lower wing on aerodynamic coefficients of biplane cellule                         | Biplane cellule     | Closed | $l_l = l_u$                            | 0-40°            |
|       |                                                                                                                                 | Isolated upper wing | Closed |                                        | 0-40°            |
|       |                                                                                                                                 | Isolated lower wing | --     |                                        | --               |
|       |                                                                                                                                 |                     |        |                                        |                  |
| 3     | Determination of effect of simultaneous flap deflection on upper and lower wings on aerodynamic coefficients of biplane cellule | Biplane cellule     | Closed | $l_l = l_u$                            | 0-40°            |
|       |                                                                                                                                 | Isolated upper wing | Closed |                                        | 0-40°            |
|       |                                                                                                                                 | Isolated lower wing | --     |                                        | --               |
|       |                                                                                                                                 |                     |        |                                        |                  |

TABLE 2

| Group | Problem                                                                                                                                                          | Study Configuration | Profile and chord length of lower wing | Position of lower wing | Flap De-flection |
|-------|------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------|----------------------------------------|------------------------|------------------|
| 4     | Determination of effect of flap deflection on coefficients of biplane cellule, with equal chords and lower wing positions. Slot closed. Basis: load measurements | Biplane cellule     | NACA 2418<br>$l_1 = 0.75 l_u$          | <u>Forward</u><br>Rear | 0-35°            |
|       |                                                                                                                                                                  | Isolated upper wing |                                        |                        | 0-35°            |
|       |                                                                                                                                                                  | Isolated lower wing | NACA 2418<br>$l_1 = 0.75 l_u$          | <u>Forward</u><br>Rear |                  |
|       |                                                                                                                                                                  | Biplane cellule     | NACA 2424<br>$l_1 = 0.5 l_u$           | <u>Forward</u><br>Rear | 0-35°            |
|       |                                                                                                                                                                  | Isolated upper wing |                                        |                        | 0-35°            |
|       |                                                                                                                                                                  | Isolated lower wing | NACA 2424<br>$l_1 = 0.75 l_u$          | <u>Forward</u><br>Rear |                  |
| 5     | Determination of effect of flap deflection on upper wing on coefficient of lower wing. Slot closed. Basis: pressure distribution measurements                    | Biplane cellule     | NACA 2418<br>$l_1 = 0.75 l_u$          | <u>Forward</u><br>Rear | 0-35°            |
|       |                                                                                                                                                                  | Isolated upper wing |                                        |                        | 0-35°            |
|       |                                                                                                                                                                  | Isolated lower wing | NACA 2418<br>$l_1 = 0.75 l_u$          | <u>Forward</u><br>Rear |                  |
|       |                                                                                                                                                                  | Biplane cellule     | NACA 2424<br>$l_1 = 0.5 l_u$           | <u>Forward</u><br>Rear | 0-35°            |
|       |                                                                                                                                                                  | Isolated upper wing |                                        |                        | 0-35°            |
|       |                                                                                                                                                                  | Isolated lower wing | NACA 2424<br>$l_1 = 0.5 l_u$           | <u>Forward</u><br>Rear |                  |

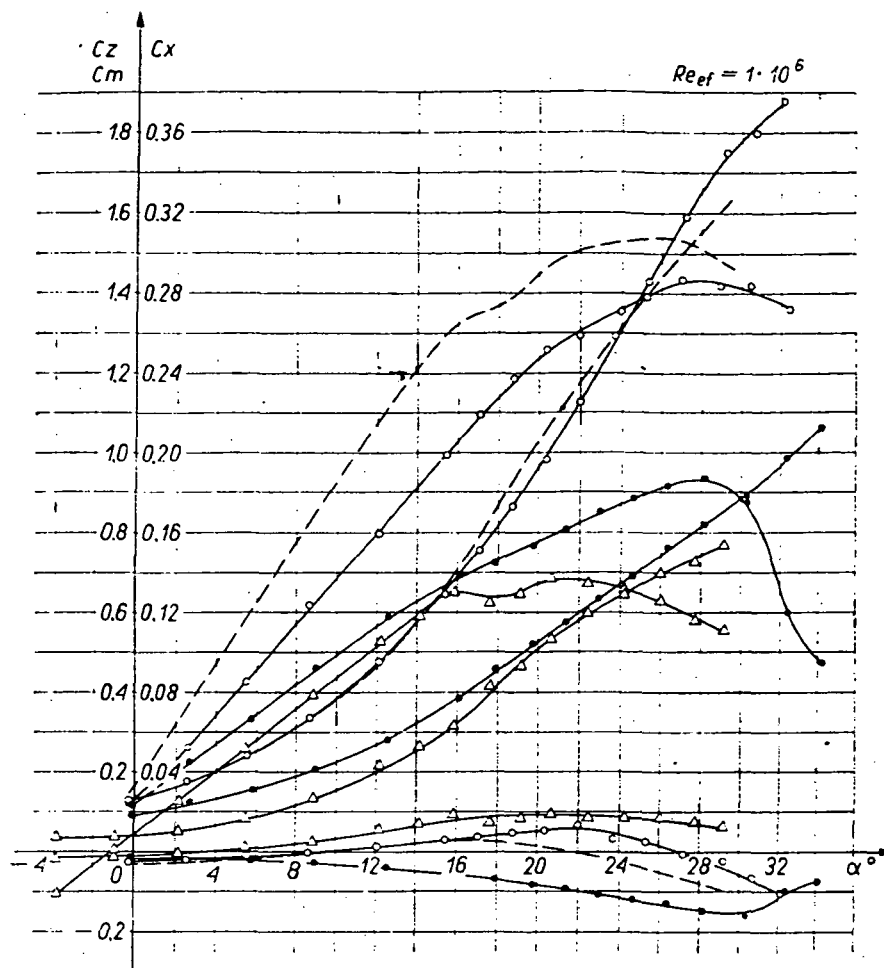


Fig. 2. Comparison of biplane cellule coefficients with isolated wing coefficients; basic flange panels; open slot;  $\delta_k = 0^\circ$ ;

- 0 -- biplane cellule, measurement no. 5508 + 5427,
- ● -- isolated upper wing, measurement no. 5515,
- Δ -- isolated lower wing, measurement no. 5506,
- isolated upper wing + isolated lower wing

Note: all the coefficients refer to the biplane surface  
 $S = 0.5408 \text{ m}^2$

The sum of the isolated upper and lower wing coefficients establishes the value of the aerodynamic coefficient of the biplane cellule without regard to interference.

On the diagram in Fig. 2 are given as an example the study results for one of the biplane-cellule configurations studied.

On this diagram are the aerodynamic lift coefficients  $C_z$ , forward resistance  $C_x$ , and bending moment  $C_m$ , depending on the angle of attack  $\alpha$

$$\left. \begin{array}{l} C_z = f(\alpha) \\ C_x = f(\alpha) \\ C_m = f(\alpha) \end{array} \right\} \text{ for } \delta_k = \text{const}$$

They are found on a combined diagram for the biplane-cellule configurations studied and for the isolated upper and lower wings corresponding to the given configuration. On these diagrams are likewise traced (with dashed lines) the dependency curve for the aerodynamic coefficients for the biplane cellule without regard to the effect of mutual operation of the wings. This curve represents the already-mentioned sum of corresponding aerodynamic coefficients for isolated wings corresponding to the given biplane configuration. The difference between the corresponding sums of aerodynamic coefficients for isolated wings and the aerodynamic coefficients for the biplane cell obtained immediately from the measurements quantitatively determined the mutual operation of the wings in the biplane cellule. /13

All the measurements were done for a speed of  $v = 30$  m/sec ( $q = 56.2$  kg/m<sup>2</sup>) in the aerodynamic tunnel, which permitted, for a wing chord profile of  $l_u = 338$  mm and a turbulence coefficient of  $TF = 1.425$ , us to obtain the effective Reynolds number  $Re_{ef} = 10^6$ .

As the surface reference for determining the aerodynamic coefficients, the biplane surface was adopted in all cases, and in the case of departure from this basis in report [16] the adopted surface reference is indicated on the diagrams.

The length of the upper wing chord was taken as the reference length for the coefficient of the bending moment.



The point with regard to which the bending moment was computed was determined with respect to the upper wing's attacking edge for the following coordinates (corresponding to the position of the aircraft's center of gravity):  $x = 68.5$  mm,  $z = 161.7$  mm.

The angles of attack of the model studied for the biplane cellule were determined with respect to the chord profile of the upper wing. In cases in which this basis was departed from in report [16], there was correspondingly indicated on the diagrams the origin of numbering for the angle of attack.

## 2.5. Analysis of Study Results

### 2.5.1. Relationships Determining the Basis for Analysis of the Study Results

The basic relationships which were taken for doing the analysis of study results and extraction of pertinent conclusions are the increments in lift coefficients for the biplane cellule as a function of angle of attack

$$\Delta C_{z_{int}} = f(\alpha) \text{ for } \delta_k = \text{const} \quad (2.1)$$

and the increments in lift coefficients as a function of flap deflection

$$C'_{z_{int}} = (\delta_k) \text{ for } \alpha = \text{const} \quad (2.2)$$

The increments (2.1) and (2.2) are the result of the mutual operation of the wings and have negative values, and therefore denote a decrease in lift force for isolated wings.

Function (2.1) was determined on the basis of the corresponding measurements as a difference of the sum of the coefficients for the isolated wing and biplane cellule.

Function (2.2) was, on the other hand, determined on the basis of corresponding diagrams on which the relationships (2.1) were given as the difference of the increments of lift coefficients for deflected and undeflected flap.

For the analysis of the study results obtained on the basis of the corresponding load measurements, The change in lift coefficient for the biplane flap was also taken, depending on the angle of flap deflection

$$C_z = f(\delta_k) \text{ for } \alpha = \text{const} \quad (2.3)$$

and its increment

$$\Delta C_z = f(\delta_k) \text{ for } \alpha = \text{const} \quad (2.4)$$

The increments, characterizing the effect of flap deflection in a biplane cellule, were determined as the difference of lift coefficient values for deflected and undeflected flap. The relationships (2.2) and (2.4) were determined in the analysis presented for the study results for an angle of attack  $\alpha = 5^\circ$ , but there is, moreover, a similar course for all the angle of attack values, for which the function  $C_z = f(\alpha)$  also has a linear course, for angles of attack smaller than those at which flow detachment occurs at the profile. /14

In addition, for the analysis of the study results, the maximum values were taken for the lift coefficients and lift increment for the biplane cellule, which are expressed, depending on the flap deflection, as

$$C_{z_{\max}} = f(\delta_k) \quad (2.5)$$

$$\Delta C_{z_{\max}} = f(\delta_k) \quad (2.6)$$

On the basis of pressure-distribution measurements, the values of the lift coefficient were determined as a function of the angle of attack for the lower-wing profile operating in the biplane cellule

$$C_z = f(\alpha) \text{ for } \delta_k = \text{const} \quad (2.7)$$

and for the isolated wing profile for various flap deflections on the upper wing. On the basis of the rest of these relationships, the relationships of increments in the lift coefficient (2.1) were determined next for the lower-wing profile.

With the increments in lift coefficients (2.1), determined on the basis of load measurements for the biplane cellule and pressure-distribution measurements for the lower wing with a difference in these values, increments in lift coefficient (2.1) were determined for the lower wing, caused by the effect of the lower wing for various deflections of the flap located on the upper wing

In order to determine the effect of the edge panels on the elongation value of the biplane cellule and the corresponding wings in the biplane cellule, the basic criterion assumed was the fixed effective elongation  $\lambda_{ef}$ . The amount of effective elongation was determined as the derivative function  $\frac{dC_x}{dC_z^2}$  in a simple linear segment of the relationship

$$C_x = f(C_z^2) \quad (2.8)$$

namely:

$$\lambda_{ef} \sim \frac{1}{\frac{dC_x}{dC_z^2}} \cdot \text{st} \quad (2.9)$$

The amount of effective elongation during the use of edge panels in the case of the studies presented was: for the biplane cellule  $\lambda_{efk} = 3$ , for the isolated upper wing,  $\lambda_{efu} = 5.65$ , for the isolated lower wing,  $\lambda_{efl} = 5.41$ .

2.5.2. Effect of flap deflection on the upper or lower wing and the effect of simultaneous flap deflection on both wings on the increments in lift coefficients for the biplane cellule

Effect on  $\Delta C_{z_{int}} = f(\alpha)$  for  $\delta_k = \text{constant}$ .

The basis for establishing the effect of flap deflection on the increments in lift coefficient is the analysis of the function relation (2.1) shown in Figs. 3 and 4.

Fig. 3 presents for comparison the increments (2.1) for the biplane cellule, for various flap deflections at the upper or lower wing.

Fig. 4 presents the increments (2.1) for the biplane cellule caused by mutual operation of the wings for simultaneous flap deflection at both wings.

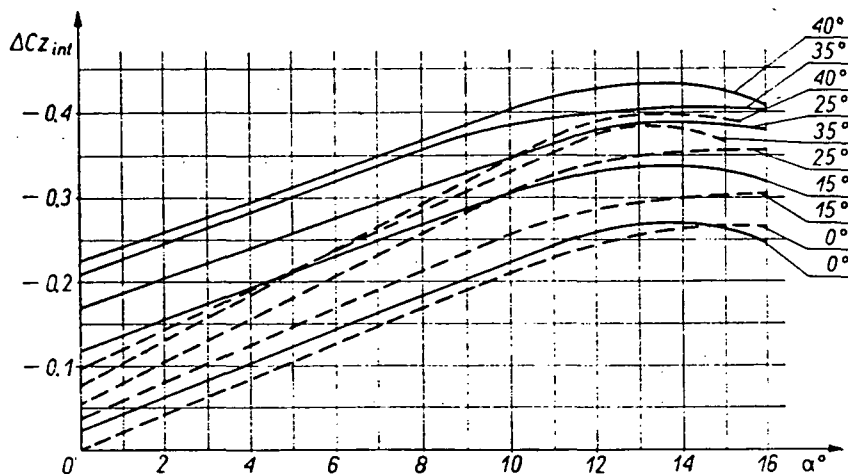


Fig. 3. Comparison of increments in lift coefficient in the biplane cellule, caused by mutual operation of the wings for flap deflection on the upper or lower wing;

———— flap deflected on upper wing  $\delta_{ku}$   
 ----- flap deflected on upper wing  $\delta_{kl}$

From a consideration of the above-mentioned diagrams, the following observations result:

-- the increments  $\Delta C_{z_{int}}$  for flap deflection at the

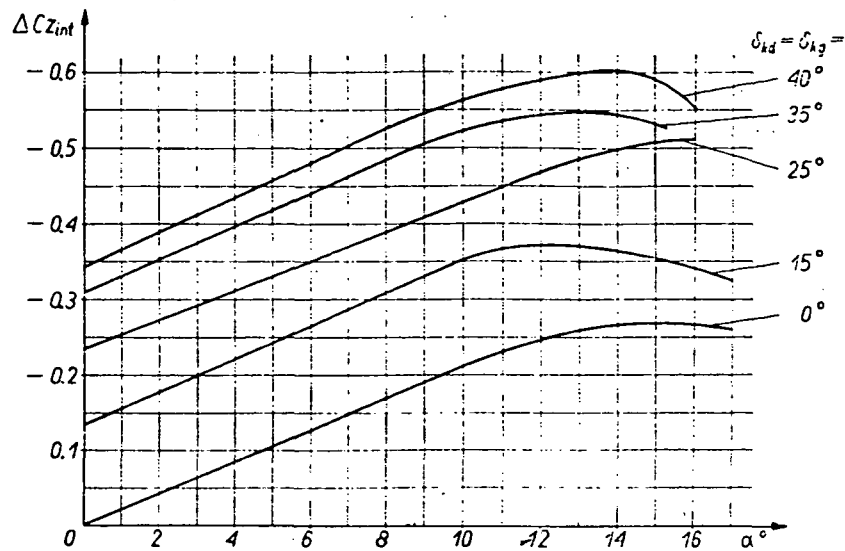


Fig. 4. Comparison of increments in lift coefficient  $\Delta C'z_{int}$  in the biplane cell, caused by mutual operation of the wings, depending on the angle of attack for simultaneous flap deflection on both wings at the same angle  $\delta_{ku} = \delta_{kl}$ .

lower wing are less with reference to absolute value than increments for flap deflections at the upper wing. From a comparison of the relationship in Fig. 2, moreover, it results that the differences in absolute values of these increments are greater for small angles of attack and decrease with an increase in the angle of attack. In comparison with the relationship  $\Delta C'z_{int}$  in Fig. 3, the conclusion is drawn that the mutual effect of the wings during flap deflection at the lower wing is less than during flap deflection at the upper wing. Likewise, associated with this, decreases in lift force are smaller;

/16

--the absolute values of the increments  $\Delta C'z_{int}$  which occur during simultaneous flap deflection on both wings (Fig. 4) are greater than increments during flap deflection on only one wing. The conclusion from this is that during simultaneous flap deflection at both wings, greater lift forces occur, and therefore decreases in lift force, caused by the mutual operation of the wings, are likewise greater.

Effect on  $\Delta C'z_{int} = f(\lambda_k)$  at  $\alpha = \text{constant}$ .

Analysis of the functional relationship of the increments in lift force (2.2) presented in Fig. 5 made possible the determination of the effect of flap deflection on the size of these increments for the biplane cellule.

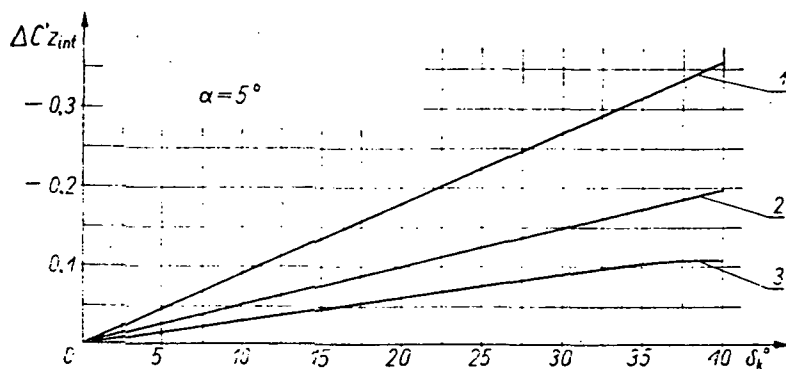


Fig. 5. Comparison of increments in the coefficient  $\Delta C'z_{int}$  in the biplane cellule, caused by the mutual operation of the wings, depending on the angle of attack; 1 - flap deflected simultaneously on upper and lower wing,  $\delta_{ku} = \delta_{kl}$ , 2 - flap deflected only on upper wing,  $\delta_{ku}$ , 3 - flap deflected only on lower wing,  $\delta_{kl}$ .

From a consideration of the above-mentioned function relationships, the following observations result:

-- the absolute values of the increments  $\Delta C'z_{int}$  depending on the flap deflection at the lower wing, are less than the increments for flap deflection at the upper wing. The conclusion drawn from this is that the operation of the wings and the associated decreases in lift force is less in the case of flap deflection at the lower wing than in the case for flap deflection at the upper wing;

-- the absolute values of the increments  $\Delta C'z_{int}$  depending on the simultaneous deflection of flaps at the same angle on both wings are somewhat greater than the sum of the increments  $\Delta C'z_{int}$  for flap deflection separately, i.e. flap deflection for the upper or lower wing alone. It is possible to explain this by the fact that such large values of the lift force occur during simultaneous flap deflection on both wings and, in connection with this, by the occurrence of large decreases in the lift force, caused by mutual operation of the wings.

Effect on  $\Delta C_z = f(\lambda_k)$  at  $\alpha = \text{constant}$

In order to determine the effect of flap deflection on increments in the lift coefficient, the function relation (2.4) presented in Fig. 6 was analyzed. This effect was determined at  $\alpha = 5^\circ$ , but as has been already explained, the conclusions drawn are correct within the limits of a linear relationship between the lift coefficient and the angle of attack.

From a consideration of the above-mentioned function relationships, the following observations result:

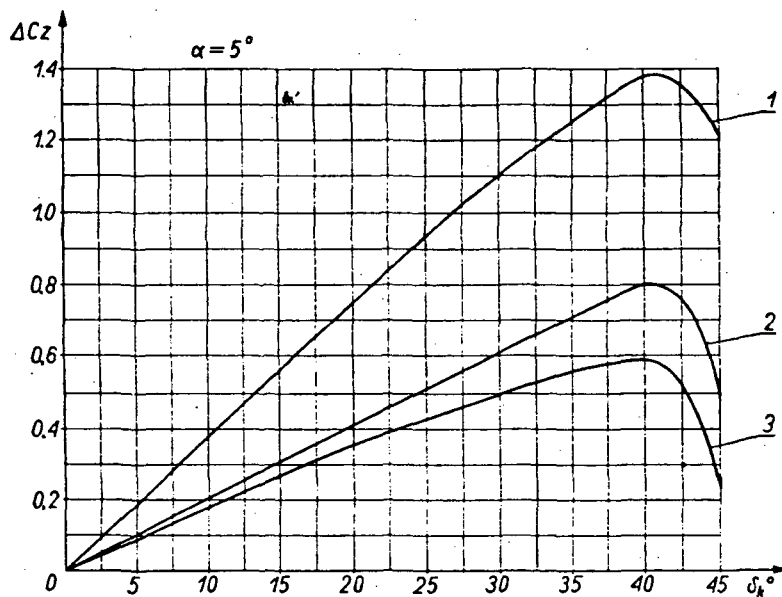


Fig. 6. Comparison of increments in lift coefficient  $\Delta C_z$  of the biplane cellule, depending on the angle  $\delta_k$  of flap deflection;

1 - flap deflected simultaneously at the upper and lower wing,

$\delta_{ku} = \delta_{kl}$ ,

2 - flap deflected only at the lower wing,  $\delta_{kl}$ ,

3 - flap deflected only at the upper wing,  $\delta_{ku}$

-- flap deflection at the lower wing permitted us to obtain higher values for the increments  $\Delta C_z$ , in comparison with values which we obtained for flap deflection at the upper wing. This is caused by a smaller decrease in lift force, especially on the lower wing. The decrease in lift force at the lower wing, for flap deflection at the upper wing, will be discussed in particular in point 2.5.4; /17

-- simultaneous flap deflection on both wings causes the increments  $\Delta C_z$ , whose numerical values are, however, somewhat smaller than the sum of the increments which were obtained for flap deflection at only

the upper or lower wing. This conclusion is associated with the conclusion obtained for the increments  $\Delta C_{z_{int}}$ . An increase for the

lift force for the biplane cell as a result of flap deflection at both wings causes an increase in the mutual operation of the wings and in the associated absolute values of the increments  $\Delta C_{z_{int}}$ , and likewise an increase in the increments  $\Delta C_z$ .

$$\text{Effect on } \Delta C_{z_{max}} = f(\lambda_k)$$

On the basis of analysis of the function relationship (2.6) of the increments in lift coefficient given in Fig. 7, the effect of flap deflection was determined on this relationship for the biplane cellule.

On the basis of the analysis of the relationship (2.6), given in Fig. 8, the effect of an open slot was determined on this relationship for flap deflection at the upper wing.

From a consideration of the function relations mentioned above, the following observations result:

-- maximum values of increments in the coefficient  $\Delta C_{z_{max}}$  are greater for a flap deflected on the lower wing than for the upper wing. Decidedly greater are the values  $\Delta C_{z_{max}}$  for simultaneous flap deflection at both wings in comparison with values obtained for a flap deflected only at the upper or lower wing. This is caused by the effect of mutual operation of the wings, which in this case retards flow detachment during streamlining of the biplane profile;

--opening the slot causes an increase in the values of the increments  $\Delta C_{z_{max}}$ , during which, with an increase in flap deflection of the upper wing, we obtained these values as greater than the values obtained for an undeflected flap. This is caused by the mutual operation of the wings, which, in this case also, retards flow detachment during streamlining of the biplane profile.

/18



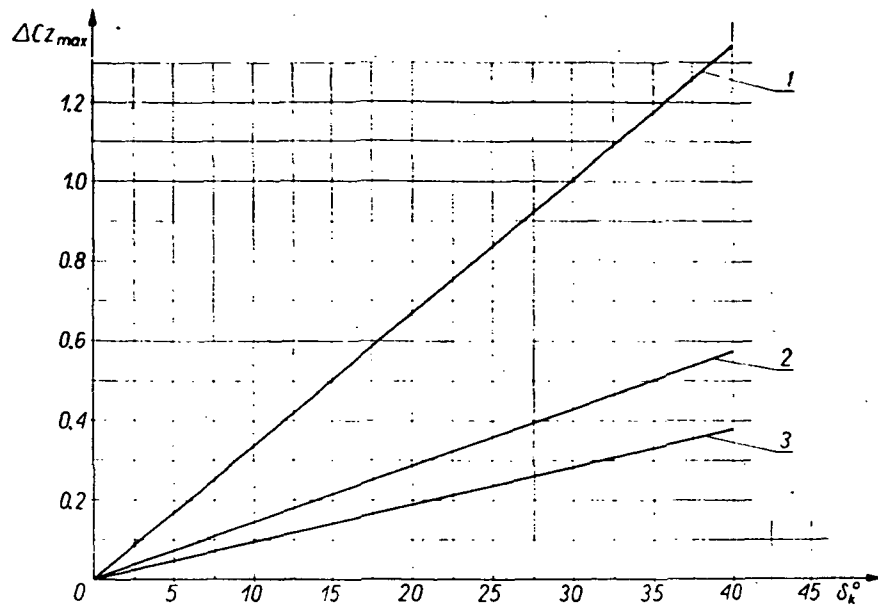


Fig. 7. Comparison of increments in maximum lift coefficient  $\Delta C_{z_{max}}$  in biplane cellule, depending on the angle  $\delta_k^\circ$  of flap deflection;

- 1 - flaps deflected simultaneously on upper and lower wings,  $\delta_{ku} = \delta_{kl}$ ,
- 2 - flap deflected only at the lower wing,  $\delta_{kl}$ ,
- 3 - flap deflected only at upper wing,  $\delta_{ku}$ .

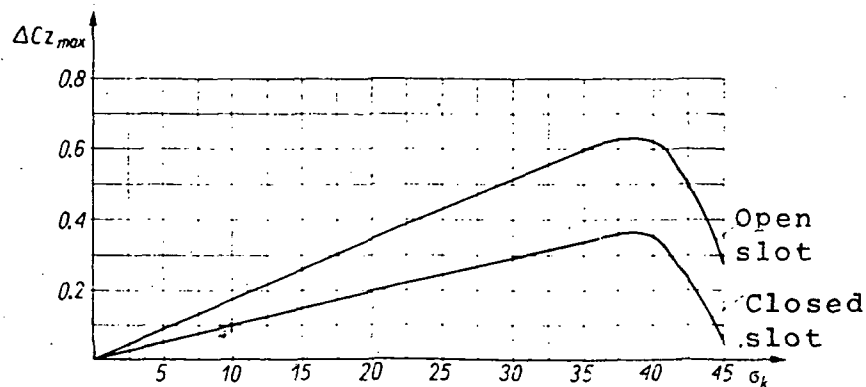


Fig. 8. Comparison of increments in the maximum lift coefficient  $\Delta C_{z_{max}}$ , depending on the angle of flap deflection  $\delta_k$  at the upper wing for closed and open slots.

2.5.3. Effect of reduced chord length and changes in the position of the lower wing on the aerodynamic coefficients of the biplane cellule with deflected flap on the upper wing.

The effect of reducing the chord length of the lower wing on the aerodynamic coefficients of the biplane cellule with flap deflection at the upper wing was determined on the basis of the analysis of the relationships (2.1), (2.2), (2.6), and (2.7), presented in Figs. 8-11. These relationships are given for a lower-wing chord length  $l_1 = l_u$ ,  $l_1 = 0.75 l_u$ , and  $l_1 = 0.5 l_u$  for two positions of the lower wing with respect to the upper, namely for the forward and rear positions.

From a comparison of the above-mentioned function relationships, the following observations result:

-- the increments  $\Delta C_{z_{int}}$ , depending on the angle of attack (2.2) (Figs. 19, 20, and 21), decrease their absolute value with a decrease in lower-wing chord length. The most favorable position of the lower wing, however, is the forward position, because the increments  $\Delta C_{z_{int}}$  are then less than increments obtained for the rear position. The smallest increments  $\Delta C_{z_{int}}$  with reference to absolute values were obtained for a lower wing of  $l_1 = 0.5 l_u$  in the forward position;

-- the increments  $\Delta C_{z_{int}}$ , depending on the flap deflection (2.2) given in Fig. 9, decrease with respect to absolute value with a decrease in lower-wing chord length. Likewise expressed is the effect on  $\Delta C'_{z_{int}}$  of the position of the lower wing with respect to the upper. This effect is particularly great for the lower wing with reduced chords (Fig. 10). The smallest absolute values of increments  $\Delta C_z$  were obtained for the lower wing with chord length  $l_1 = 0.5 l_u$  in the forward position;

-- values of the increments depending on flap deflection (2.4), given in Fig. 11, increase for decreasing lower-wing chord length. Likewise evident is the favorable effect of the lower-wing position relative to the upper wing. The largest values

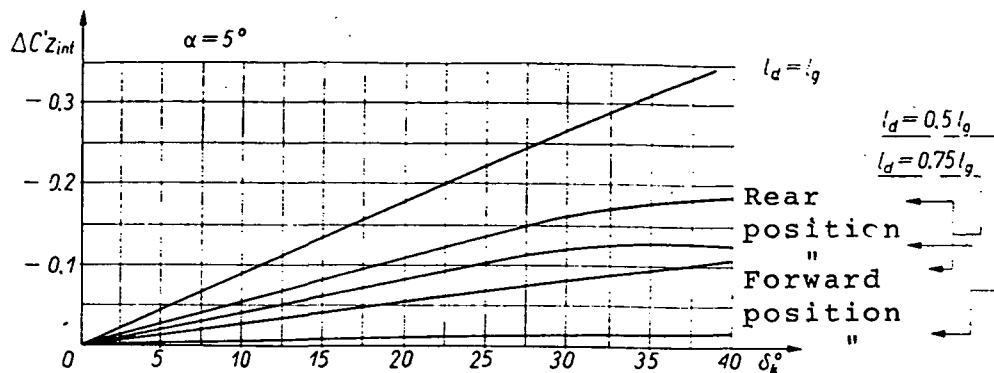


Fig. 9. Comparison of increments in the lift coefficient  $\Delta C'_{z_{int}}$  in a biplane cellule, caused by the mutual operation of the wings, depending on flap deflection at the upper wing for a reduced lower-wing chord in forward and rear position

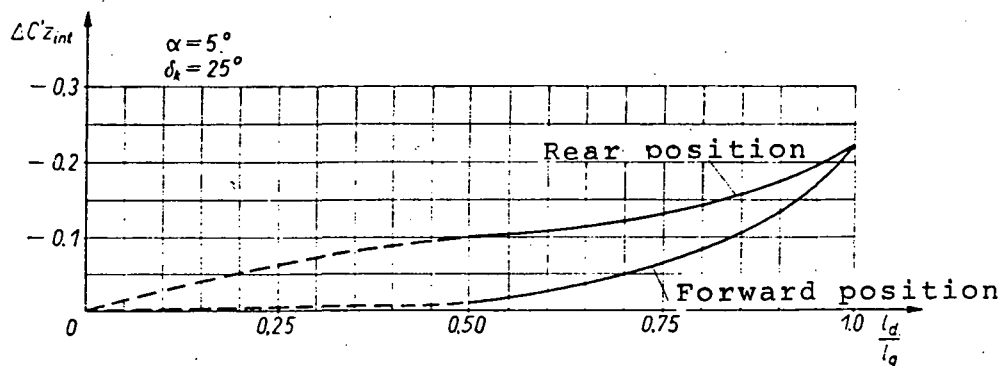


Fig. 10. Effect of lower-wing chord length on the increments in lift coefficient  $\Delta C'_{z_{int}}$ , caused by mutual operation of the wings for forward and rear positions of the lower wing.

for the increments  $\Delta C_z$  were obtained for a lower wing with chord length  $l_1 = 0.5 l_u$  in the forward position; /20

-- values for increments  $\Delta C_{z_{max}}$  depending on flap deflection (2.6), given in Fig. 12, increase with decreasing lower-wing chord length. Higher values for these quantities were obtained for the lower wing in the forward position. The highest

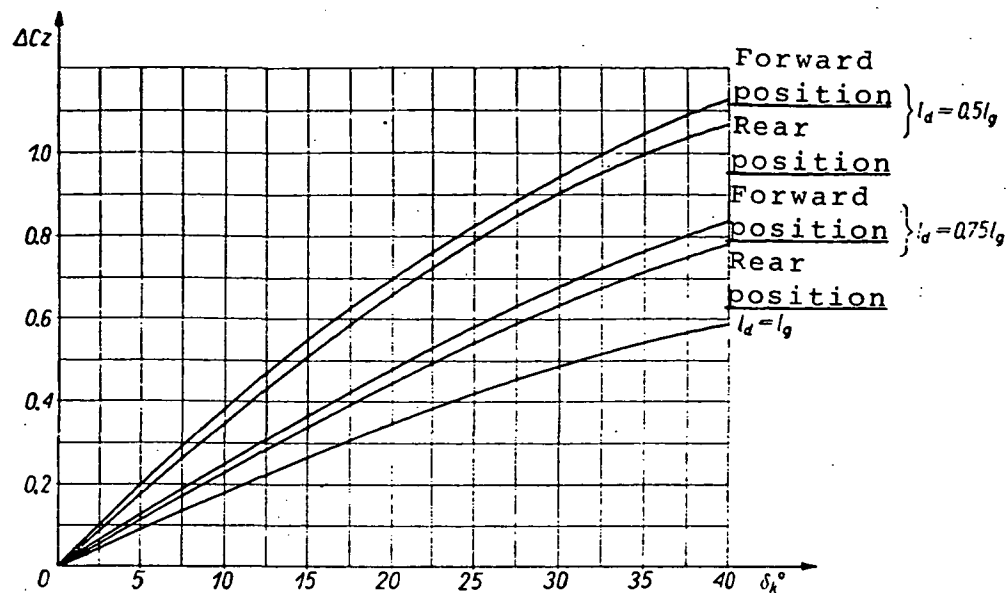


Fig. 11. Increments in lift coefficient  $\Delta C_z$  of the biplane cellule, depending on angle  $\delta_k$  of flap deflection. Lower wing with same chord length in forward and rear position,  $\alpha = 5^\circ$ .

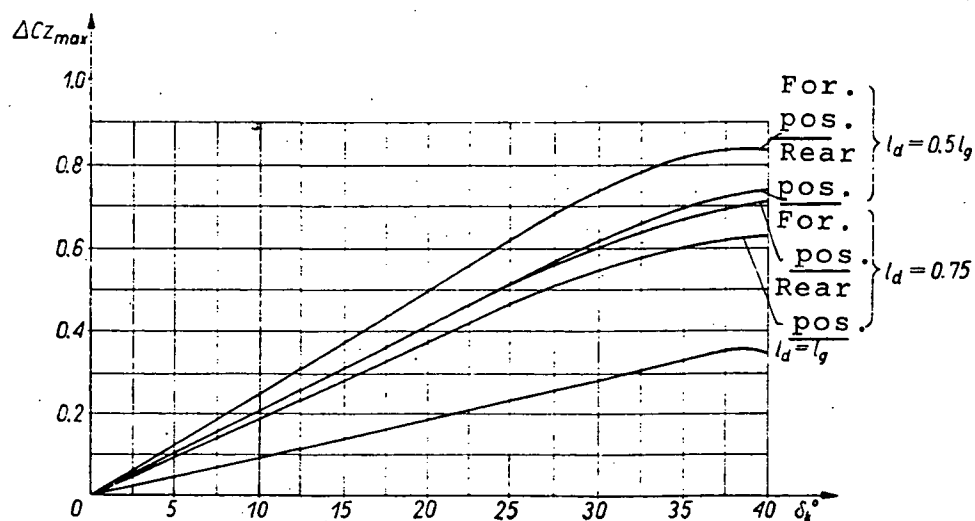


Fig. 12. Comparison of increments in maximum lift coefficient  $\Delta C_{z_{max}}$  for biplane cellule, depending on angle  $\delta_k$  of deflection; slot closed, lower wing with chord  $l_l = 0.75 l_u$  and  $l_l = 0.5 l_u$  in operation of the wings, depending on flap deflection at the upper wing for a reduced lower-wing chord in forward and rear position.

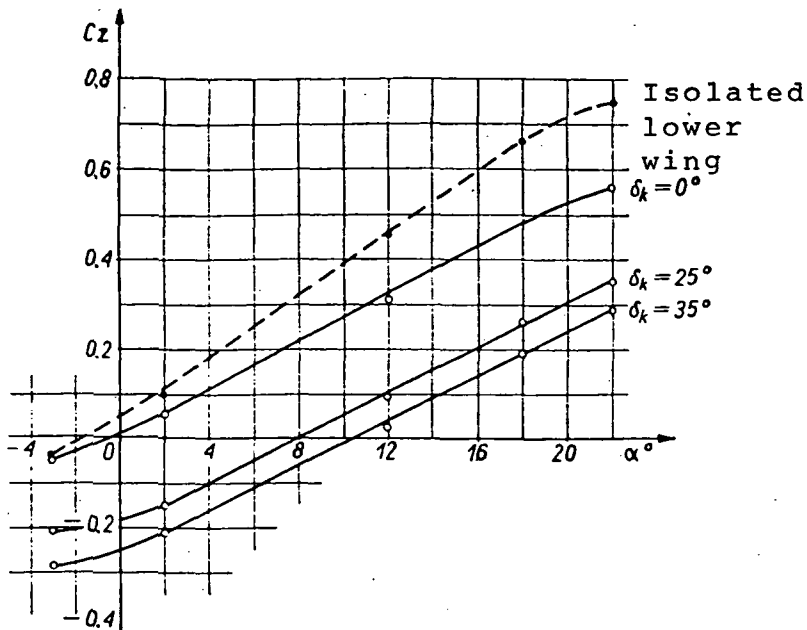


Fig. 13. Lift coefficient for lower-wing profile of biplane cellule. Lower wing with chord length  $l_1 = l_u$ .

-- 0 -- lower wing in biplane cellule

-- ● -- isolated lower wing

Lower wing profile P-II 14,  $\alpha^\circ$  = angle of attack measured with respect to upper wing chord,  $C_z$  = coefficient related to surface of biplane  $S_1 = 0.5408 \text{ m}^2$

Note: slot and flap found only on upper wing

values of these quantities were obtained for a lower wing of  $l_1 = 0.5 l_u$  in the forward position;

-- all the changes analyzed above for increments in the lift coefficient for the biplane cellule, at various chord lengths and lower-wing positions caused a change in the amount of mutual operation of the wings.

In reducing the chord length of the lower wing, the mutual operation of the wings is decreased and in association with this, favorable changes occur in the values of the increment relationships for the lift coefficient for the biplane cellule.

In case of reduction of the lower-wing chord length, one ought also to obtain its corresponding position relative to the upper wing, for in the opposite case one cannot obtain the expected favorable changes in the lift-coefficient increments.

#### 2.5.4. Effect of flap deflection at the upper wing on lift coefficients of the lower wing, based on measurements of pressure distribution.

The basis for establishing the effect of flap deflection on the lift coefficients of the lower wing with reduction of the

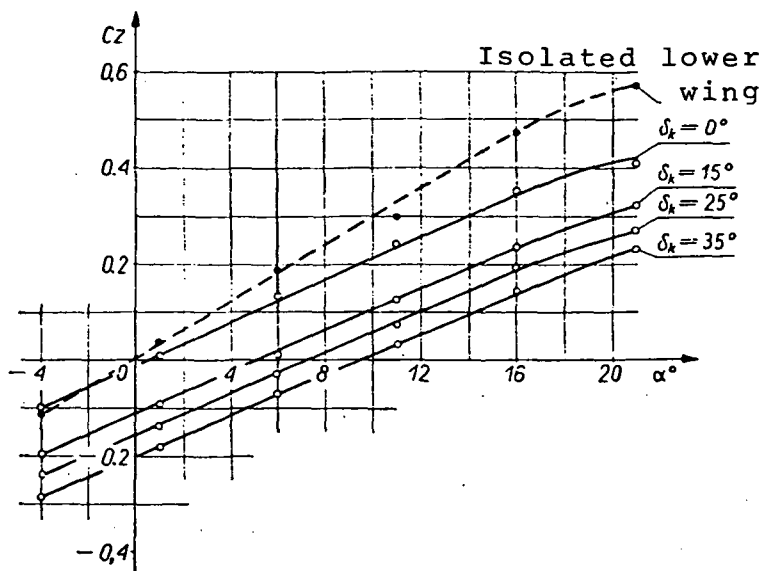


Fig. 14. Lift coefficient of lower-wing profile in biplane cellule.

Lower wing with chord length  $l_1 = 0.75 l_u$  in forward position;  
 — 0 — lower wing in biplane cellule

— ● — isolated lower wing  
 Lower wing profile NACA 2418,  
 $\alpha$  = angle of attack measured with respect to upper wing chord,  $C_z$  = coefficient relative to the surface of the biplane  $S_1 = 0.473 \text{ m}^2$ .

Note: slot and flap found only on upper wing

lower-wing chord length in forward and rear positions of the lower wing is the analysis of the function relationship (2.7) determined on the basis of pressure-distribution measurements on the lower-wing profile and presented in Figs. 13-17.

Based on the diagrams mentioned above, the increments in lift coefficient  $\Delta C_{z_{int}}$  were determined next for the lower wing, and were caused by the operation of the upper wing, which is given as an example in Figs. 18 and 19.

In addition, subtracting correspondingly the increments  $\Delta C_{z_{int}}$  obtained

for the lower wing from pressure-distribution measurements caused by the operation of the upper wing, from the increments for a biplane unit caused by mutual operation of the wings, the increments in lift were determined for the upper wing which were caused by the operation of the lower wing. These increments are also given as an example in Figs. 18 and 19.

Having determined the lift coefficients depending on the angle of attack for the isolated upper wing and the increments  $\Delta C_{z_{int}}$ , and intermediate mode was determined for the isolated upper wing (Figs. 18 and 19), and then the diagrams of the function

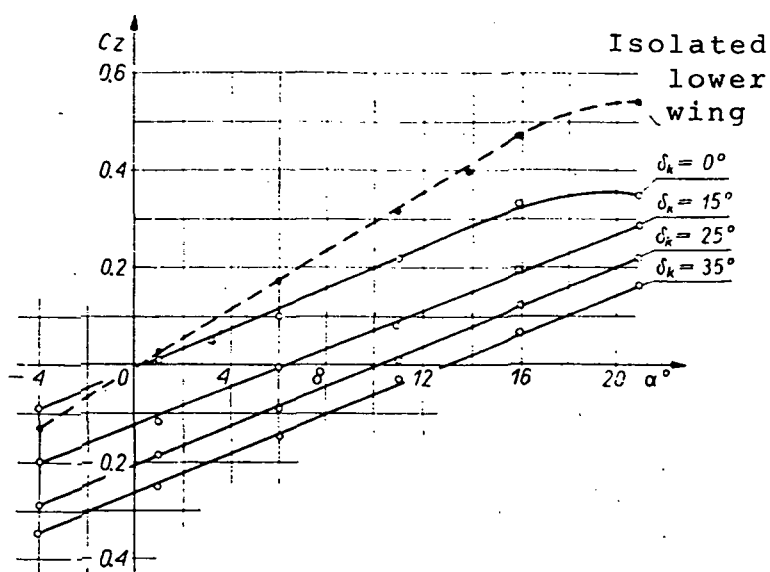


Fig. 15. Lift coefficient for profile of lower wing in biplane cell. Lower wing with chord length  $l_1 = 0.75 l_u$  in rear position

— 0 — lower wing in biplane cellule

-- ● -- isolated lower wing

Lower wing with profile NACA 2418,  $\alpha$  = angle of attack measured relative to chord of upper wing,  $C_z$  = coefficient relative to the surface of the biplane  $S_1 = 0.473 \text{ m}^2$

Note: slot and flap found only on upper wing

chord length  $l_1 = 0.75 l_u$ , in the forward position:

for  $\delta_k = 25^\circ$   $\epsilon = 7^\circ$   
 $\delta_k = 35^\circ$   $\epsilon = 9.5^\circ$

in the rear position:

for  $\delta_k = 25^\circ$   $\epsilon = 10^\circ$   
 $\delta_k = 35^\circ$   $\epsilon = 13^\circ$

for lower wing with chord length  $l_1 = 0.5 l_u$  in the forward position:

for  $\delta_k = 25^\circ$   $\epsilon = 5.5^\circ$   
 $\delta_k = 35^\circ$   $\epsilon = 7.5^\circ$

in the rear position:

for  $\delta_k = 25^\circ$   $\epsilon = 9.5^\circ$   
 $\delta_k = 35^\circ$   $\epsilon = 12.5^\circ$

relationship (2.7) for the upper wing were drawn. These relationships were given as an example in Figs. 20 and 21.

From a consideration of /24 the above-mentioned function relationships, the following observations result:

-- the dependency curves for the lift coefficient (2.7) for the lower wing for flap deflection are shifted in parallel in the direction of the positive values for the angle of attack, so that the lift coefficients are equal to zero in the range of the angles of attack used, namely:

for the lower wing with chord length  $l_1 = l_u$ , there was obtained:

for  $\delta_k = 25^\circ$   $\epsilon = 7.5^\circ$ ;  
for the lower wing with

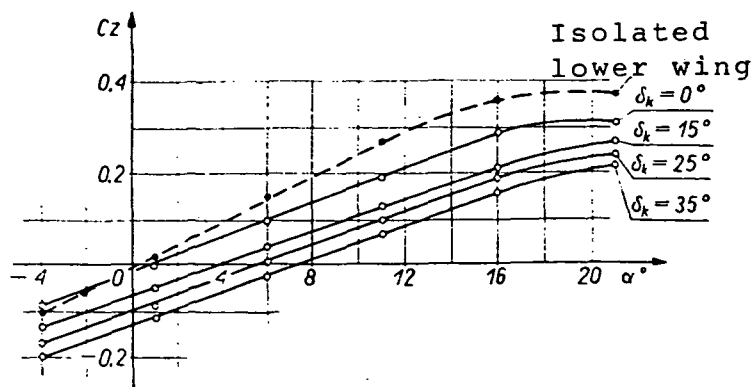


Fig. 16. Lift coefficient of lower-wing profile for biplane unit. Lower wing with chord length  $l_1 = 0.5 l_u$  in forward position

— 0 — lower wing in biplane cellule

-- ● -- isolated lower wing

Lower wing with profile NACA 2424,  $\alpha$  = angle of attack measured relative to upper-wing chord,  $C_z$  = coefficient relative to surface of biplane  $S_1 = 0.405 \text{ m}^2$ .

Note: slot and flap found only in upper wing

On the basis of the values quoted for the angles of attack corresponding to zero values of the lift force, it is possible to conclude that reduction of the chord length of the lower wing causes a decrease in the value  $\epsilon^\circ$  for the forward position of the lower wing. On the other hand, for the rear position of the wing with a reduction of chord length for the lower wing, an increase occurs in the value of  $\epsilon^\circ$ . This conclusion once again confirms the necessity for selecting a corresponding position for the lower wing relative to

the upper;

--absolute values of the increments  $\Delta C_{z_{int}}$  for the lower wing given as an example in Figs. 18, 19, and 20, show a decrease for reduction of chord length of the lower wing, for which smaller values correspond to the forward position of the lower wing. The most favorable of the cases studied is the lower wing with chord length  $l_1 = 0.5 l_u$  in the forward position. This is caused by a lesser operation of the upper wing upon the lower one, and especially during flap deflection, and associated with this, smaller decreases occur in the lift force on the lower wing;

-- absolute values of the increments  $\Delta C_{z_{int}}$  for the upper wing and the intermediate mode which was determined and given 18 and 19 could have positive or negative values. For small angles of attack and small flap deflections, the values of  $\Delta C_{z_{int}}$  have negative values. However, with an increase in flap deflection



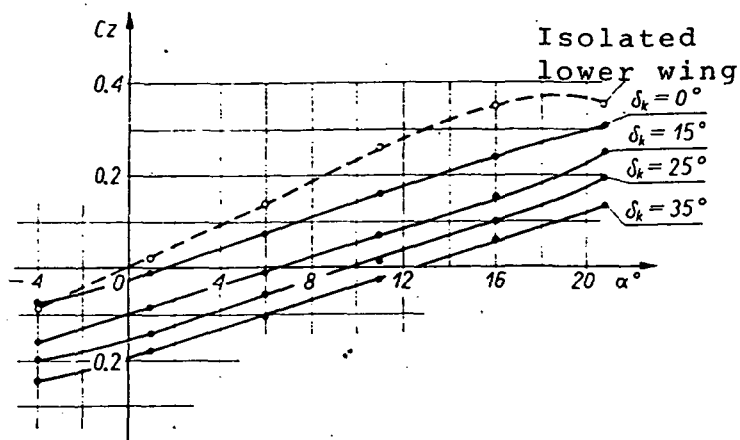


Fig. 17. Lift coefficient from pressure distribution for biplane cellule. Lower wing with chord length  $l_1 = 0.5 l_u$  in rear position

— 0 — lower wing in biplane unit

-- ● -- isolated lower wing

Lower wing with profile NACA 2424,  $\alpha$  = angle of attack measured relative to upper-wing chord,  $C_z$  = coefficient relative to surface of biplane  $S_1 = 0.405 \text{ m}^2$ .

Note: slot and flap found only on upper wing

and angle of attack, the values of the increments  $\Delta C_{z_{int}}$  decrease with respect to absolute values, and finally change sign;

--from the course of the relationship (2.1) and (2.7) for the upper wing (Figs. 18, 19, 20, and 21), it is seen that the lower wing operates upon the upper one to cause a decrease in lift force at small angles of attack and small flap deflections at the upper wing. On the other hand, for all angles of attack and flap deflections for the lower wing, it causes an increase in lift force at the upper wing.

Likewise in contrast to the conclusions which were drawn for the lower wing, it is seen that reduction of the lower-wing chord does not affect the upper wing favorably, because it causes smaller increases in lift force on the upper wing.

### 3. Conclusions

On the basis of analysis of the study results for the previously changed configurations of the biplane with flap, it is possible to draw the following conclusions concerning the effect of flap deflection on the lift coefficient.

#### 3.1. Effect of Deflection of a Flap Located on the Upper Wing

Deflection of a flap located on the upper wing may cause, in the range of angles of attack used, a large decrease in lift force

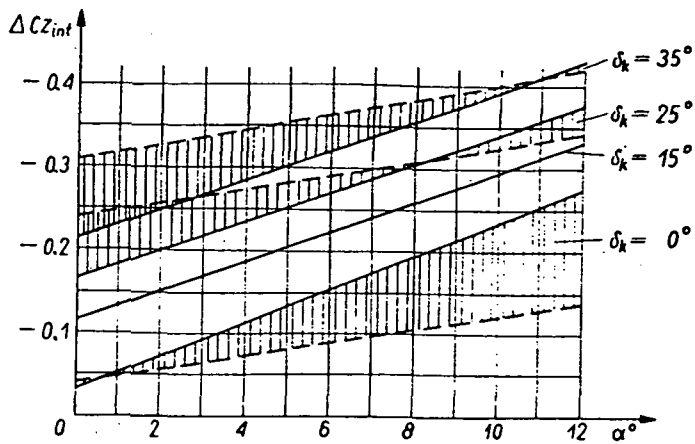


Fig. 18. Comparison of increments in the lift coefficient  $\Delta C_{z_{int}}$  in the biplane unit caused by mutual operation of the wings, depending on the angle of attack. Lower wing with profile CAGI-P-II-14 and chord length  $l_1 = 0.5 l_u$ . Flap deflected on upper wing;  
 — biplane cellule  
 --- lower wing  
 |||| upper wing

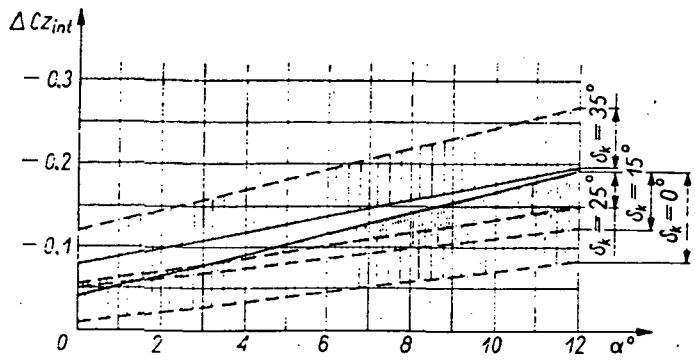


Fig. 19. Increments in lift coefficient  $\Delta C_{z_{int}}$  in biplane unit, caused by mutual operation of wings depending on angle of attack. Lower wing with profile NACA 2424 and chord length  $l_1 = 0.5 l_u$  in forward position. Flap deflected on upper wing;  
 — biplane cellule  
 --- lower wing  
 |||| upper wing

on the lower wing. For an undeflected flap or its sufficiently small deflection, there appears at the upper wing as a result of the effect of the lower wing a negative increment in lift force, which gradually changes sign from negative to positive for an increase in the angle of flap deflection. This positive increment at the upper wing is, however, of such a size that it does not compensate for the negative increments of lift force at the lower wing.

/27

### 3.2. Effect of Deflection of Flap Located at Lower Wing

Deflection of a flap located on the lower wing causes a decrease in lift force of the entire biplane due to interference, but this decrease is expressed less in comparison with the decrease in lift force which accompanies the deflection of a flap located on the upper wing. In addition, in deflecting a flap located on the lower wing, in comparison with the decrease with upper-wing flap deflection,

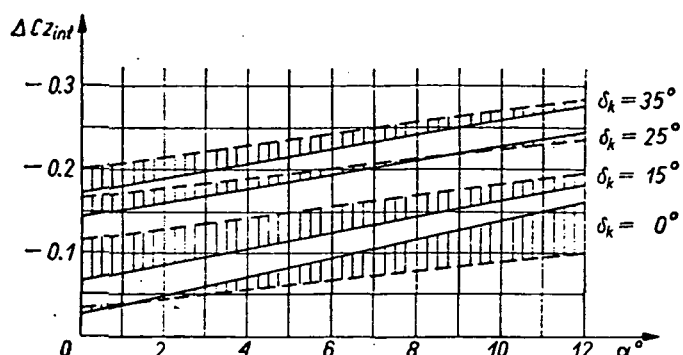


Fig. 20. Increments in lift coefficient  $\Delta C_{z_{int}}$  in biplane cellule, caused by mutual operation of the wings depending on angle of attack. Lower wing with profile NACA 2424 and chord length  $l_1 = 0.5 l_u$  in rear position. Flap deflected on upper wing;

— biplane unit  
 ---- lower wing  
 |||| upper wing

the maximum lift force coefficient  $\Delta C_{z_{max}} = f(\delta_k)$  of the biplane which are higher than the sum of increments of maximum lift coefficient caused by flap deflection at only the upper or lower wing.

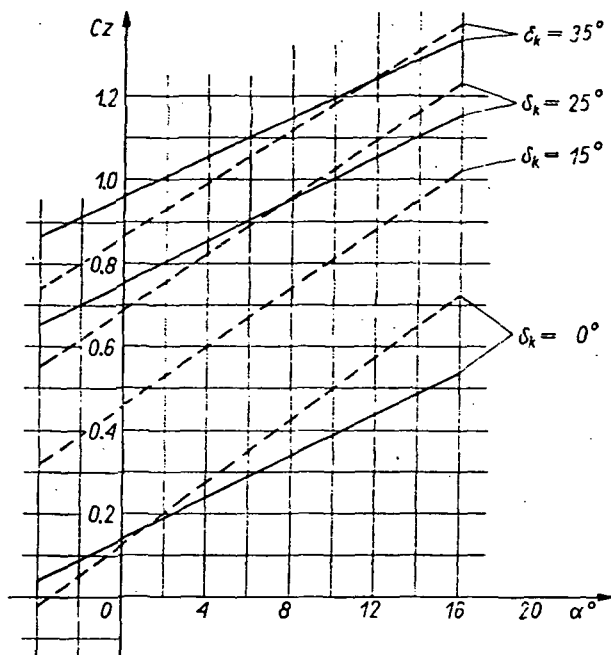
### 3.4. Effect of Reducing Lower-Wing Chord Length During Deflection of a Flap Located on the Upper Wing

In a biplane configuration with a flap located on the upper wing, it is favorable, from the aerodynamic viewpoints, to use a lower wing with chord length reduced, in comparison with the upper-wing chord. The position of the lower-wing chord in relation to the upper-wing chord must then be such that the edge of attack lies in one vertical plane (i.e. the forward position). Both these means, used together, give smaller decreases in the lift coefficient  $\Delta C_{z_{int}} = f(\alpha)$  at  $\delta_k = \text{constant}$ , caused by interference of the wings, for both the lower wing and the whole biplane cellule, in comparison with the lift coefficient for the

higher increments are obtained for the lift coefficient  $\Delta C_z = f(\delta_k)$  which depends on flap deflection and higher values for increments in the maximum lift coefficient  $\Delta C_{z_{max}} = f(\delta_k)$ .

### 3.3. Effect of Simultaneous Flap Deflection at Both Wings

Simultaneous flap deflection at the upper and lower wing is most favorable from the aerodynamic viewpoint of flap deflection at only the upper or lower wing. Among other things, values are then obtained for increments in



biplane unit with the lower-wing  
chord  $l_1 = l_u$ .

Fig. 21. Lift coefficients for  
upper wing in biplane unit.  
Lower wing with chord length  
 $l_1 = l_u$  for profile P-II-14,

----- upper wing isolated  
——— upper wing in bi-  
plane cellule,

$\alpha$  = angle of attack measured  
relative to upper-wing chord,  
 $C_z$  = coefficient relative to  
surface of biplane  $S=0.5408 \text{ m}^2$

## REFERENCES

1. Betz, A., "The opposite influence of two wings," Zeit. Flugtech. 5, 235 (1914).
2. Betz, A., "Calculation of the lift force for a biplane cellule from the specific values for single wings," T.B. 1, 103 (1917).
3. Prandtl, L., "Induced resistance of various wings," T.B. 3, 309.
4. Aerodynamik [Aerodynamics], Vol. 2, Fuchs-Hopf-Seewald, Berlin, 1934.
5. Bonder, J., "Motion of two cylinders in a perfect fluid, together with an application to near-ground flight," Warszawskie Towarzystwo Politechniczne. Sprawozdanie i Prace [Warsaw Polytechnical Society: Reports and Works], Vol. 4, Warsaw, 1925.
6. Munk, M.M., "The air forces on a systematic series of biplane and triplane cellule models," Report 256, NACA, 1927.
7. Knight, M. and Noyes, R., "Wind tunnel pressure distribution tests on a series of biplane wing models," Technical Notes 310, 325, 330, NACA, 1929.
8. Noyes, R.W., "Pressure distribution tests on a series of Clark biplane cellules with special reference to stability," Report 417, NACA, 1932.
9. Kuhn, P., "Working charts for the determination of the lift distribution between biplane wings," Report 445, NACA, 1932.
10. Diehl, W.S., "Relative loading on biplane wings," Report 458, NACA, 1933.
11. Multhopp, H., "A simple way to calculate the pressure distribution for a single wing of a biplane," Luftf. Forsch. 15, 543 (1938).
12. Küchemann, D., "Calculation of pressure distribution for a single wing of a biplane," Luftf. Forsch. 15, 543 (1938).
13. Prosnak, W.J., "Contribution to the theory of biplane wing sections," Buil. PAN.
14. Prosnak, W.J., "Remarks on approximate conformal representation of biplane wing sections," Buil. PAN 2 (1975).
15. Prosnak, W.J. and M.E. Klonowska, "Algorithms for determining the aerodynamic coefficient of a biplane," Prace IPPT-PAN [Works of the IPPT, Polish Academy of Sciences], Warsaw, 1974.

16. Stasiak, J., "Aerodynamic studies of a biplane cellule segment for the M-15 aircraft," Report 30/PA/73, Aeronautics Institute, 1973.